

The Milky Way's Supermassive Black Hole: How good a case is it?

A Challenge for Astrophysics & Philosophy of Science

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Max-Planck-Institut
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G2 / DSO 2006

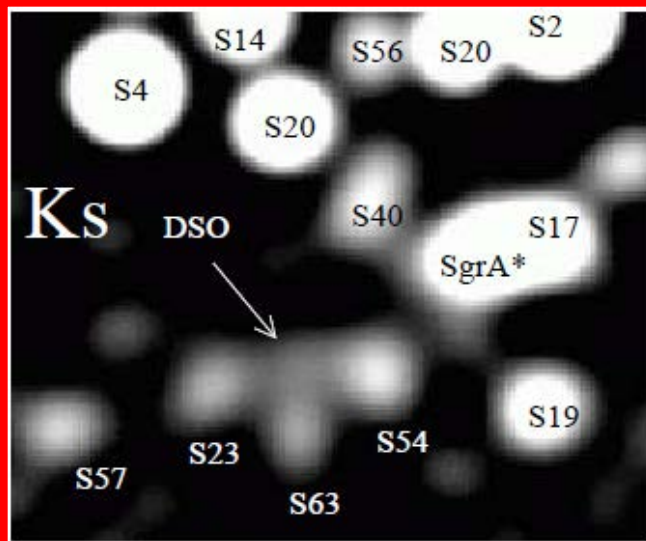


University of Cologne

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The Milky Way's Supermassive Black Hole

How good a case is it?

The Milky Way's Supermassive Black hole: How good a case is it? *A Challenge for Astrophysics & Philosophy of Science**

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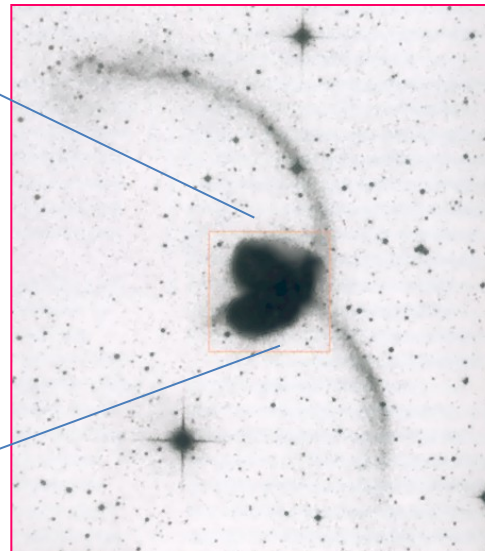
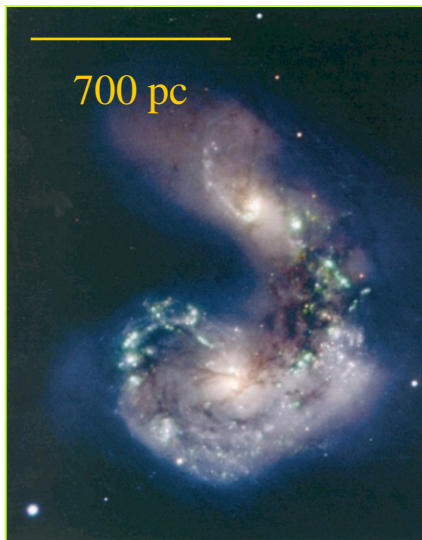
(Dated: June 1, 2016)

Foundations of Physics, 47, 553, Springer, 2017

Difference between stellar and Galactic black holes

Stellar black holes are formed through the collapse of a massive star: $M \sim 10$

Galactic black holes are formed in (together with) the central stellar cluster of massive galaxies: $M > 1,000,000$



**Antenna-Galaxy
NGC 4038/39**

20 Mpc distance
 $1'' = 140 \text{ pc}$

Investigating Supermassive Black Holes

We are actively involved in investigating SgrA* as a SMBH:

- Radio interferometric **VLBI** observations
- Infrared interferometric observations (**GRAVITY**)
- Multifrequency radio and infrared observations in parallel to the Event Horizon Telescope (**EHT**) observations

Providing SMBH relevant instrumentation, e.g.:

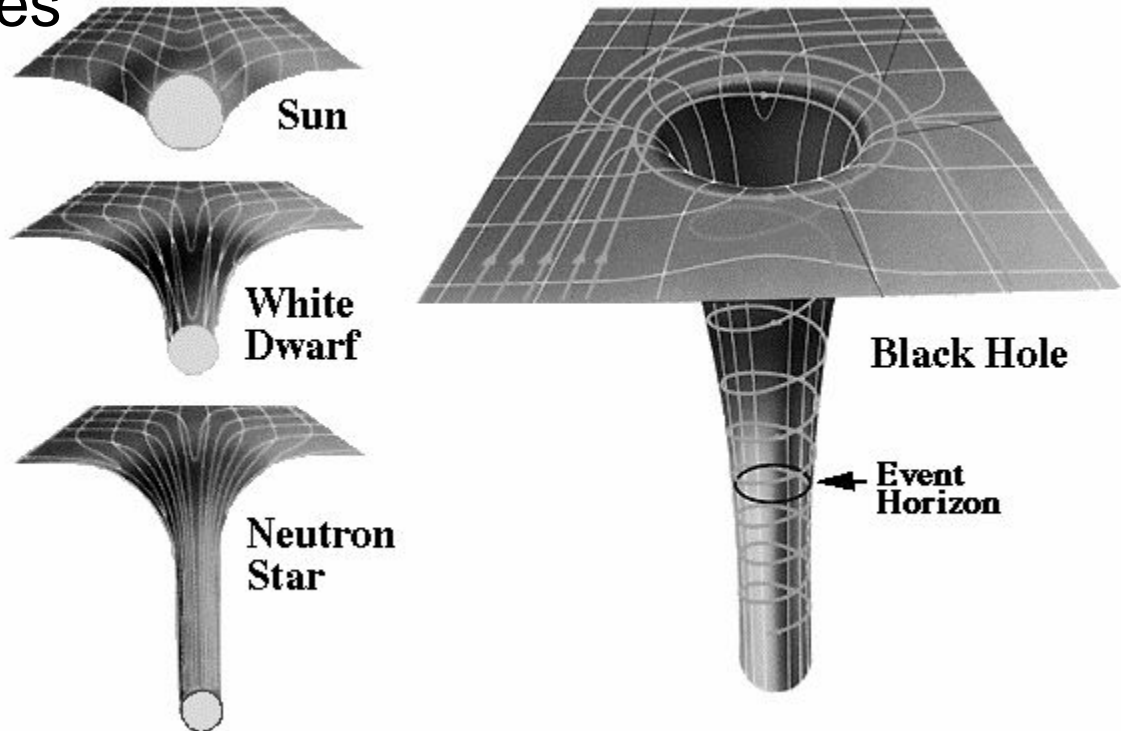
- Imaging beam combiner for the Large Binocular Telescope (**LBT**) in Arizona
- Very Large Telescope beam combiner spectrometer for the **GRAVITY** experiment
- Participation in the **MIRI** imaging spectrometer on board **JWST**

**Working definition:
What is a
(supermassive)
black hole?**

Working definition: What is a black hole?

A black hole is a geometrically defined **region of spacetime** around a **compact mass**. The gravitational is so strong that nothing can escape from inside the event horizon.

The **no-hair theorem** states that a black hole is fully described by only three externally observable classical parameters: **mass, electric charge, and angular momentum.**

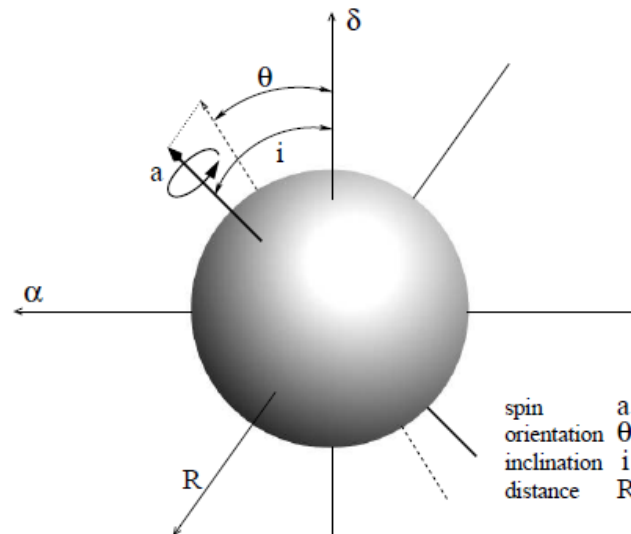


Here we suppress complications like rotation Black Holes and radiation that may come from immediate vicinity

source: <https://www.pinterest.com>

Working definition: What is a black hole?

They are characterized by an **event horizon** that, however, *cannot* become part of an external observer's past in a finite time but is an **important discriminator against other similarly compact and massive objects**.

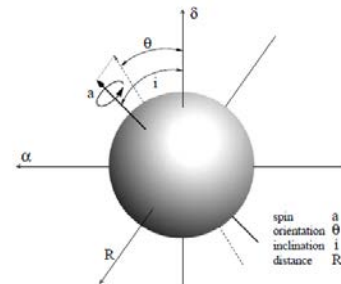
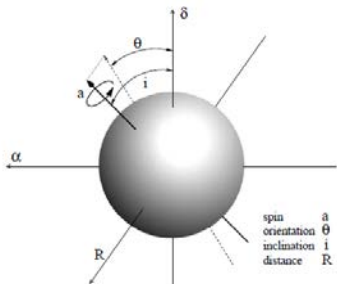


Working definition: What is a black hole?

But is the event horizon really the most adequate concept for describing observations, as indicated, for example, by the name of the project “Event Horizon Project”?

When observing a black hole such as the SMBH in the Galactic Center now, we cannot know of any amount of matter that will fall into this black hole in the future and will lead to an increase of mass and, consequently, of an increase of the size of its event horizon.

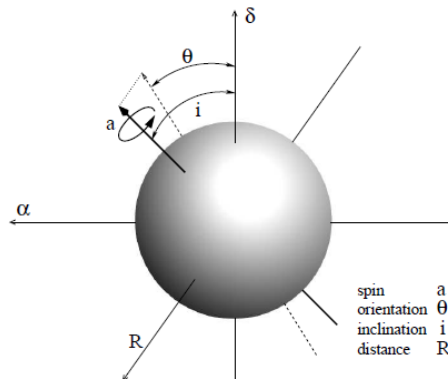
We thus need alternative notions which are of a more local nature.



Working definition: What is a black hole?

Such notions are, in fact, used. The most important one for our case is the notion of an **apparent horizon**.

For its definition, one considers **the boundary between the region where emitted light can reach infinity and the region where it cannot**. This three-dimensional boundary is called **“trapping horizon”**



Working definition: What is a black hole?

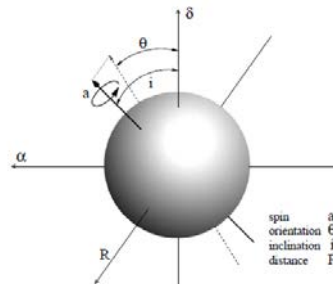
For stationary black holes of mass M_{\bullet} , the apparent horizon coincides with the (time slice of the) event horizon. In the simplest case of the Schwarzschild solution, the horizon size is given by the Schwarzschild radius

$$R_S := \frac{2GM_{\bullet}}{c^2}; \quad (1)$$

for the Kerr black hole, the horizon is located at

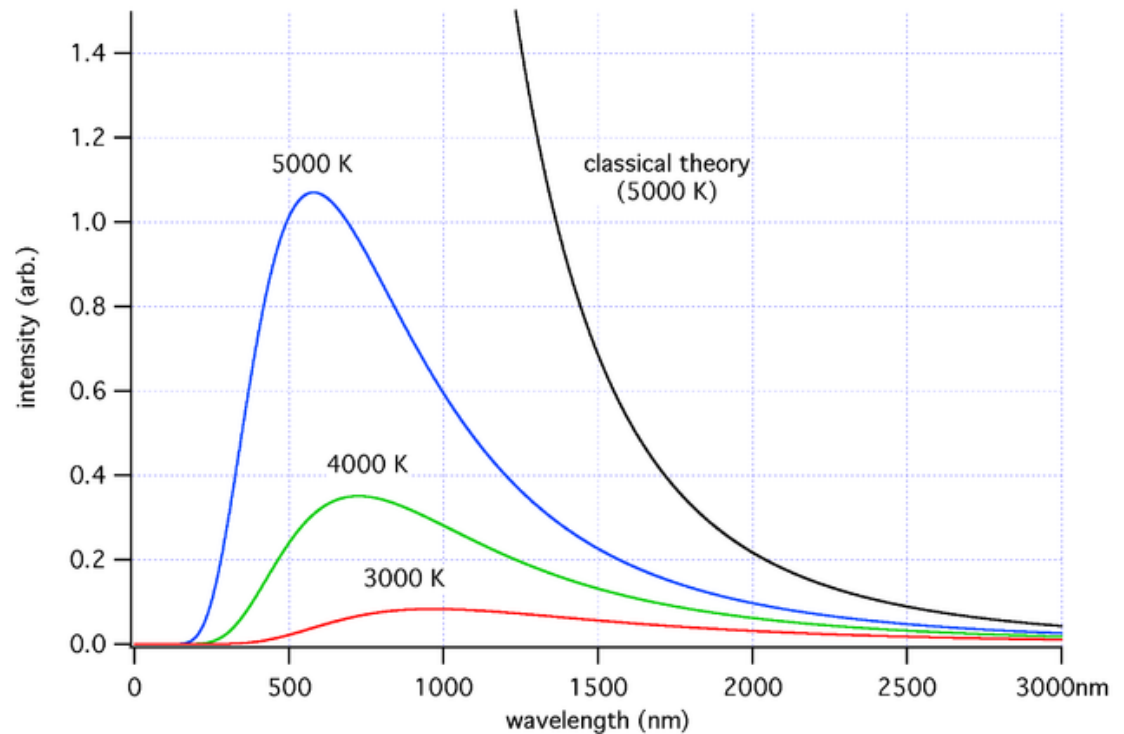
$$R_{\text{Kerr}} := \frac{GM_{\bullet}}{c^2} + \sqrt{\left(\frac{GM_{\bullet}}{c^2}\right)^2 - a^2}. \quad (2)$$

Quite generally,¹³ the apparent horizon lies *within* the event horizon or coincides with it.



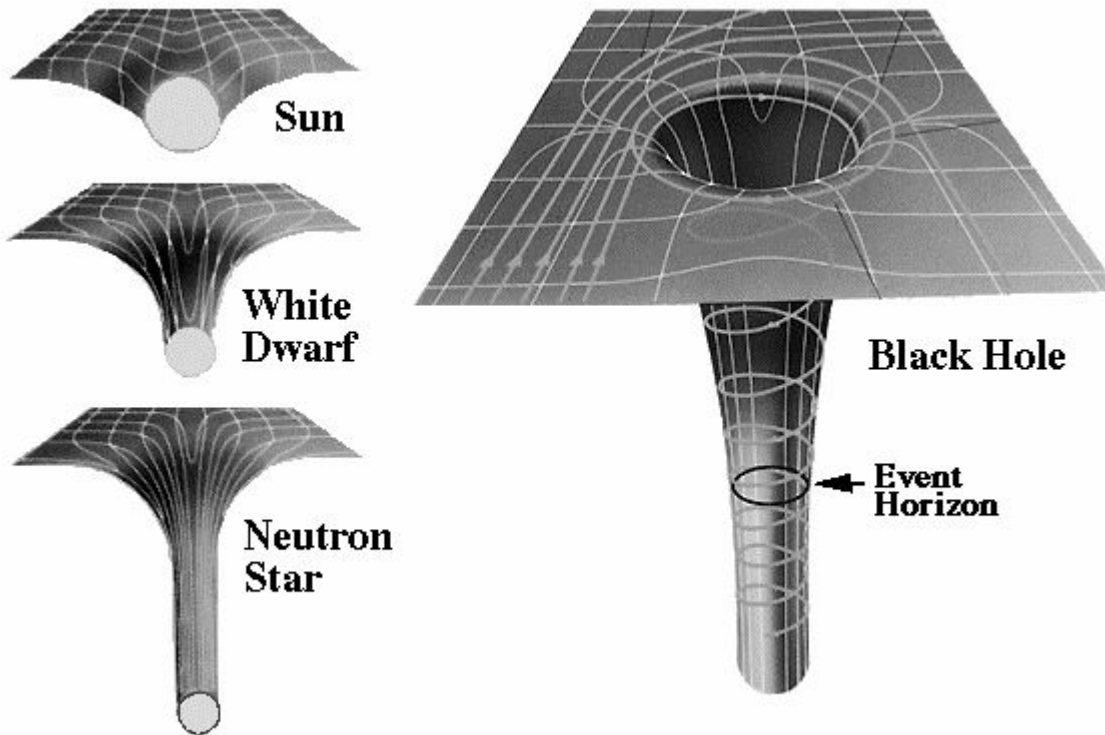
Black Body Radiation – is only relevant for micro-black holes

$$T_H = \frac{\hbar c^3}{8\pi k_B G M}$$
$$\approx 6.2 \times 10^{-14} \text{ K} \left(\frac{10^6 M_\odot}{M} \right)$$
$$\approx 6.2 \times 10^{-8} \text{ K} \left(\frac{1 M_\odot}{M} \right)$$
$$\approx 1.2 \times 10^{11} \text{ K} \left(\frac{10^{15} \text{ g}}{M} \right)$$
$$\approx 7.7 \times 10^{29} \text{ K} \left(\frac{1 \text{ TeV}}{M} \right)$$



(Hawkingstrahlung)

How can we 'proof' the existance of supermassive black holes?



Philosophical Concepts

Underdetermination

Underdetermination has a theoretical and an experimental side: The theory may not be fully complete and only highlight certain properties.

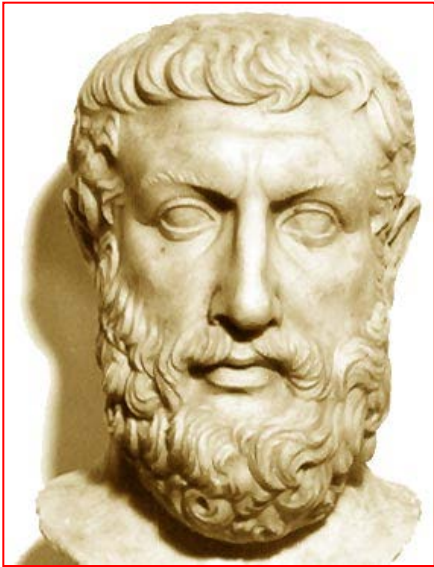
In addition, the observations may be not unique enough to clearly distinguish one possible realization of an object from another, since the interpretation of the observations may just be based on a restricted set of theoretical predictions. In the case of experiments (see e.g. Franklin, 2016; Galison, 1987), however, one has the chance to fight (i.e. minimize or even remove the effect of) underdetermination by increasing the observational evidence and combining various procedures that approach the problem with different methods or instrumental efforts.

..... and Causation

If underdetermination can be fought or even partially overcome, then causation may be used to further underline the realism or existence of an entity in a generally acceptable way. This involves the usage of a causal criterion that may be in the form of the Eleatic Principle (for a general overview see e.g. Colyvan, 1998, 2001). (Colyvan, 1998) gives a concise definition of the classical Eleatic Principle "An entity is to be counted as real if and only if it is capable of participating in causal processes".

Eleatic Principle

Named after a Greek school in lower Italy **Elea** (Ἐλέα) closely linked to the philosophers **Parmenides**, Zeno and Xenophanes of Colophon.



Parmenides

Philosophical conceptual aspect:

The School of Elea rejects any epistemological criteria based on sensual experiences. Instead they request logical standards of clarity as criteria of truth



This is how it is implemented: The Eleatic Principle or causal criterion is a test that must be passed by logical statements or objects in order to be accepted by the researchers ontology, i.e. the study of the nature of being, becoming, existence, or reality.

Historic example for such a test

Acceptance of the existence of molecules and atoms

As a further candidate procedure for sufficient evidence:

'if you can spray them, then they are real' (Hacking 1983):

If you can use entities to manipulate others, then we have sufficient evidence for their reality.

To be used as an instrument in a manipulation of other systems presupposes a quantitative precise causal profile in order to bring about the effects in question.

If the effect is successfully brought about we have sufficient evidence for the claim that there is something with this particular profile.



Realism:

Direct interaction and the possibility of repeatability and manipulation.

Anti-Realism:

The 'pure' observational nature of astrophysical research.



(Hacking 1983)

The Eleatic Principle

Colyvan's rounded out version of the Eleatic Principle

- for reasons of symmetry and theoretic virtue
- allowing for entities that are **causally idle but causally relevant**
- hence, including the Eleatic Principle relying mainly on causal entities and balancing the unsatisfactory justification attempts

Classical Eleatic Principle

as a logical test for causality that must be passed before acceptance within a scientific ontology.

The principle is mainly relying on **causally active** entities but leads to largely unsatisfactory justification attempts

Reality of mathematical sentences, physical laws etc.

Causation with objects in factual time sequence

Underdetermination and Causation

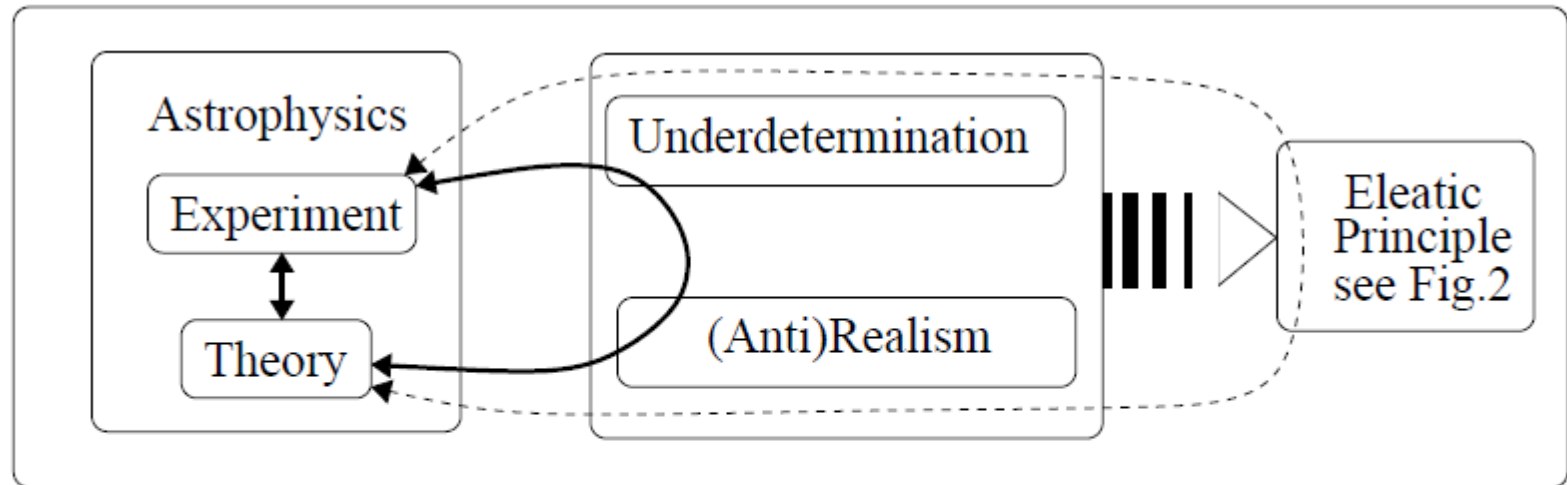


Fig. 1 Linkage between experiment and theory interpreted via the concept of realism and underdetermination finally allowing us to discuss the question of realism and existence in the framework of causation, making use of a form of the Eleatic Principle.

This structure must be filled for the Galactic Center

If underdetermination can be fought or even partially overcome, then causation may be used to further underline the realism or existence of an entity in a generally acceptable way. This involves the usage of a causal criterion that may be in the form of the Eleatic Principle (for a general overview see e.g. Colyvan, 1998, 2001). (Colyvan, 1998) gives a concise definition of the classical Eleatic Principle "An entity is to be counted as real if and only if it is capable of participating in causal processes".

Connecting Necessary and Sufficient Conditions

necessary conditions: $S \implies \forall \mu N_\mu \iff N_1 \wedge N_2 \wedge N_3 \wedge \dots \wedge N_\mu$

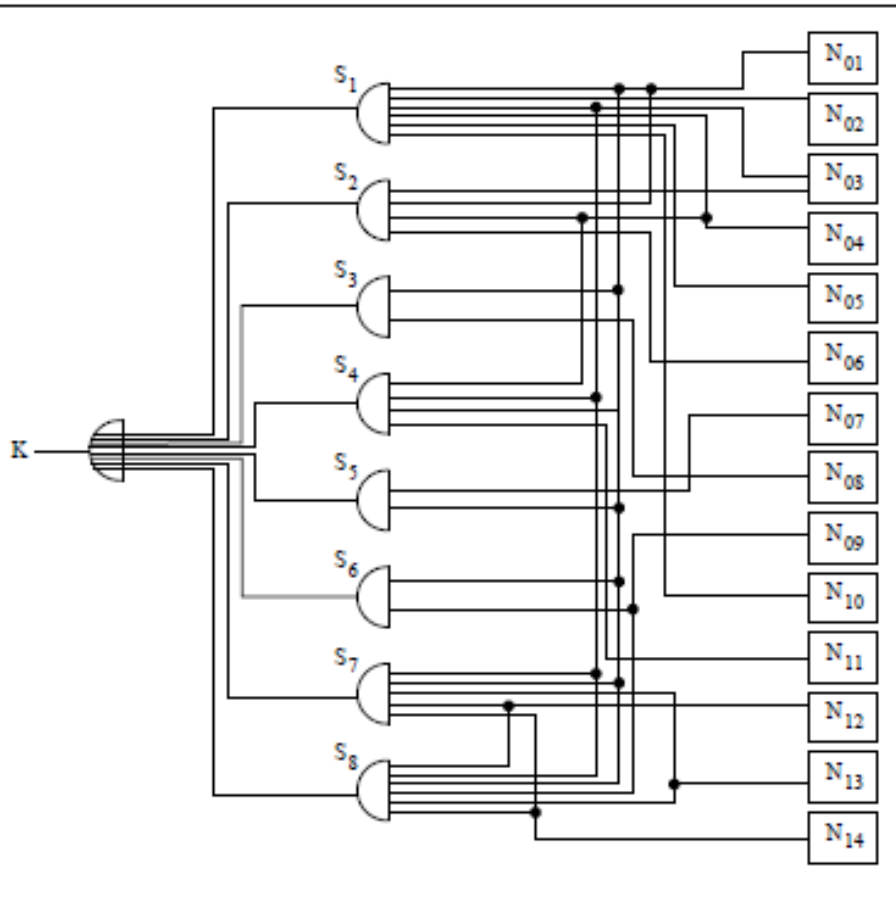
sufficient conditions: $K \implies \exists \nu S_\nu \iff S_1 \vee S_2 \vee S_3 \vee \dots \vee S_\nu$

necessary and sufficient conditions:

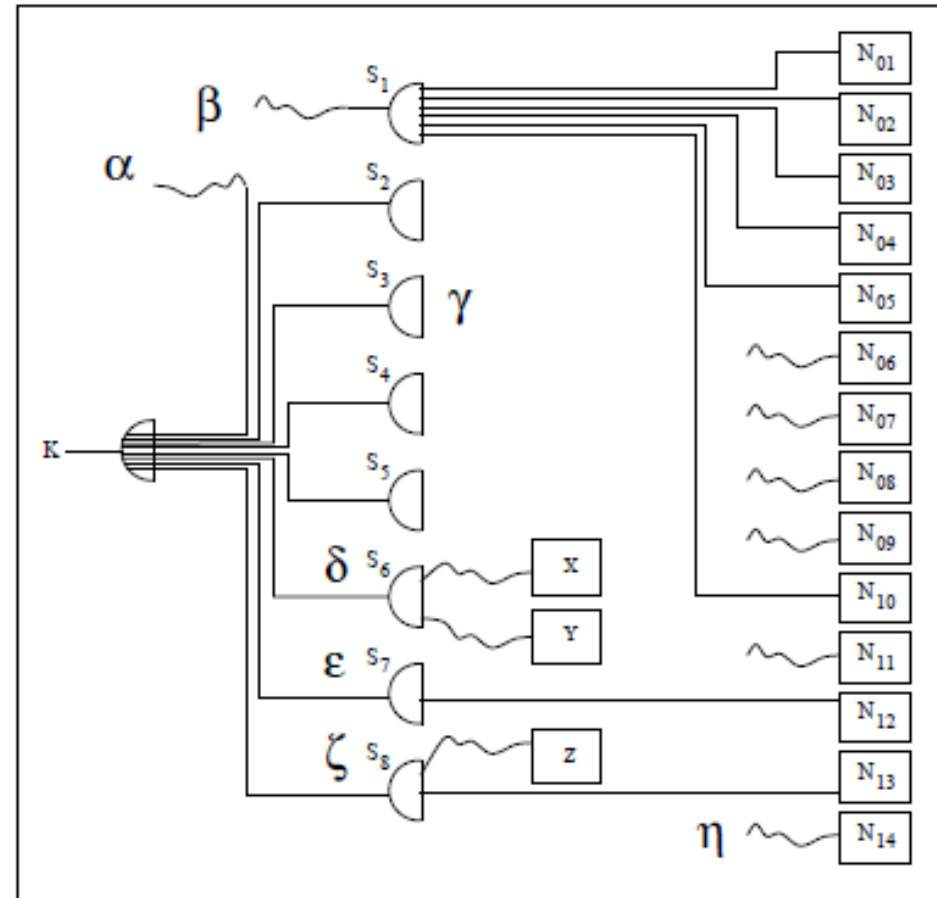
$$K \implies \exists \nu S_\nu \iff \exists \nu \forall \mu(\nu) N_{\nu, \mu(\nu)}$$

$$\begin{aligned} \exists \nu \forall \mu(\nu) N_{\nu, \mu(\nu)} \iff & (N_{\kappa_{1,1}} \wedge \dots \wedge N_{\kappa_{1, \mu(1)}})_1 \vee \\ & (N_{\kappa_{2,1}} \wedge \dots \wedge N_{\kappa_{2, \mu(1)}})_2 \vee \dots \\ & \vee (N_{\kappa_{\nu,1}} \wedge \dots \wedge N_{\kappa_{\nu, \mu(\nu)}})_\nu \end{aligned}$$

Connection necessary and sufficient conditions



Yes



No

Necessary Conditions for the presence of a Black Hole

label	necessary condition
N_1	Is object at nominal position of SgrA*?
N_2	Is size of emitting region in SgrA* sufficiently small?
N_3	Is mass of SgrA* in agreement with SMBH masses?
N_4	Does the distance to SgrA* place it at the center of the Milky Way?
N_5	Is the manipulative success for SgrA* similar to other SMBH candidates?
N_6	Is a bright fast jet originating from SgrA*?
N_7	Do we detect a merger ringing signal in gravitational waves from SgrA*?
N_8	Do we detect an exceptionally bright flare from SgrA*?
N_9	Do stars and pulsars close to SgrA* give indications for a SMBH?
N_{10}	Is the spectrum of the surroundings of SgrA* what we expect from a SMBH?
N_{11}	Do we detect a photon ring in SgrA* in addition to orbiting matter?
N_{12}	Do VLBI images of SgrA* show a shadow as expected for a SMBH?
N_{13}	Do we detect photo-center motion of SgrA* with NIR- and/or mm-radio-interferometry?
N_{14}	Can we differentiate for SgrA* between jet components and hot-spot?

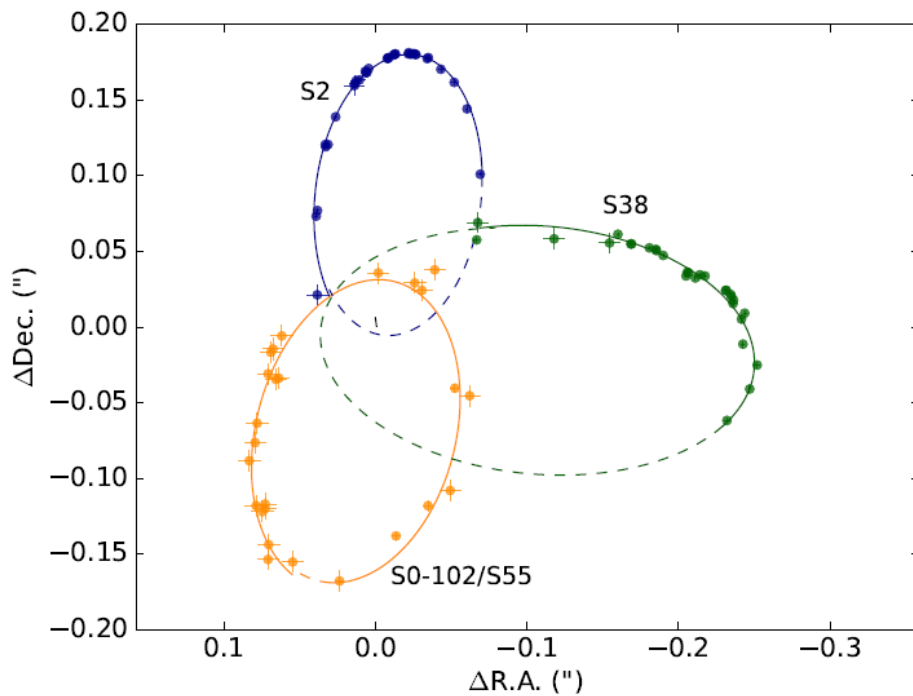
Table 3 Table of possible necessary conditions that can be combined to result in a sufficient condition required to call SgrA* a SMBH. The necessary conditions have been formulated as logical entities for which we can attribute the logical values “true” or “false” within the theoretical predictions for supermassive black holes in section 2.

Example 1

Proving that we indeed probe a
relativistic regime:

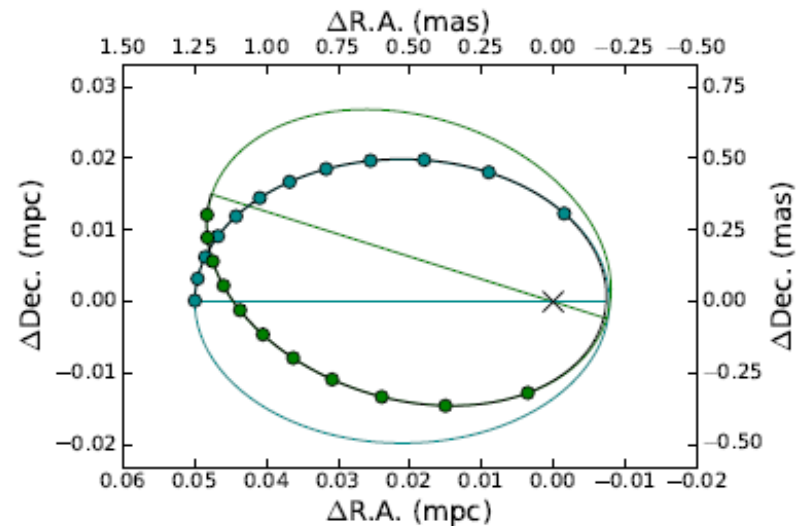
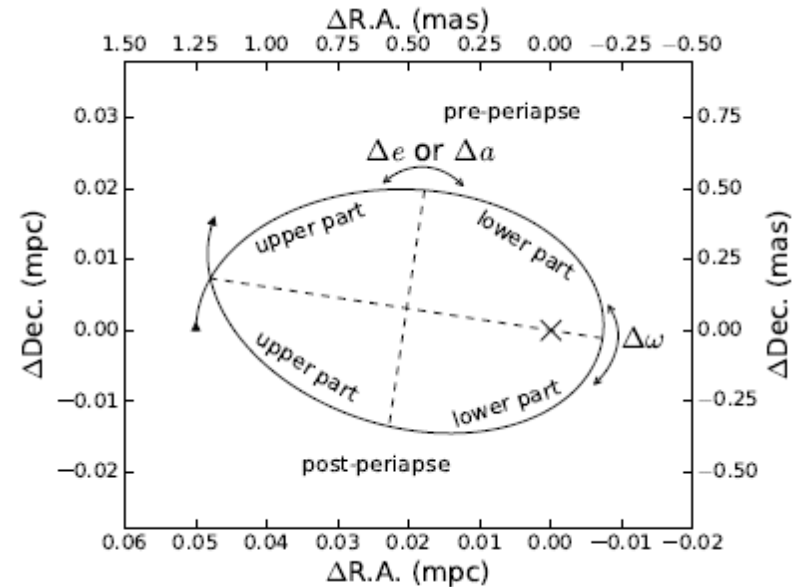
Relativistic orbits of stars

Parsa et al. 2017, ApJ 845, 22

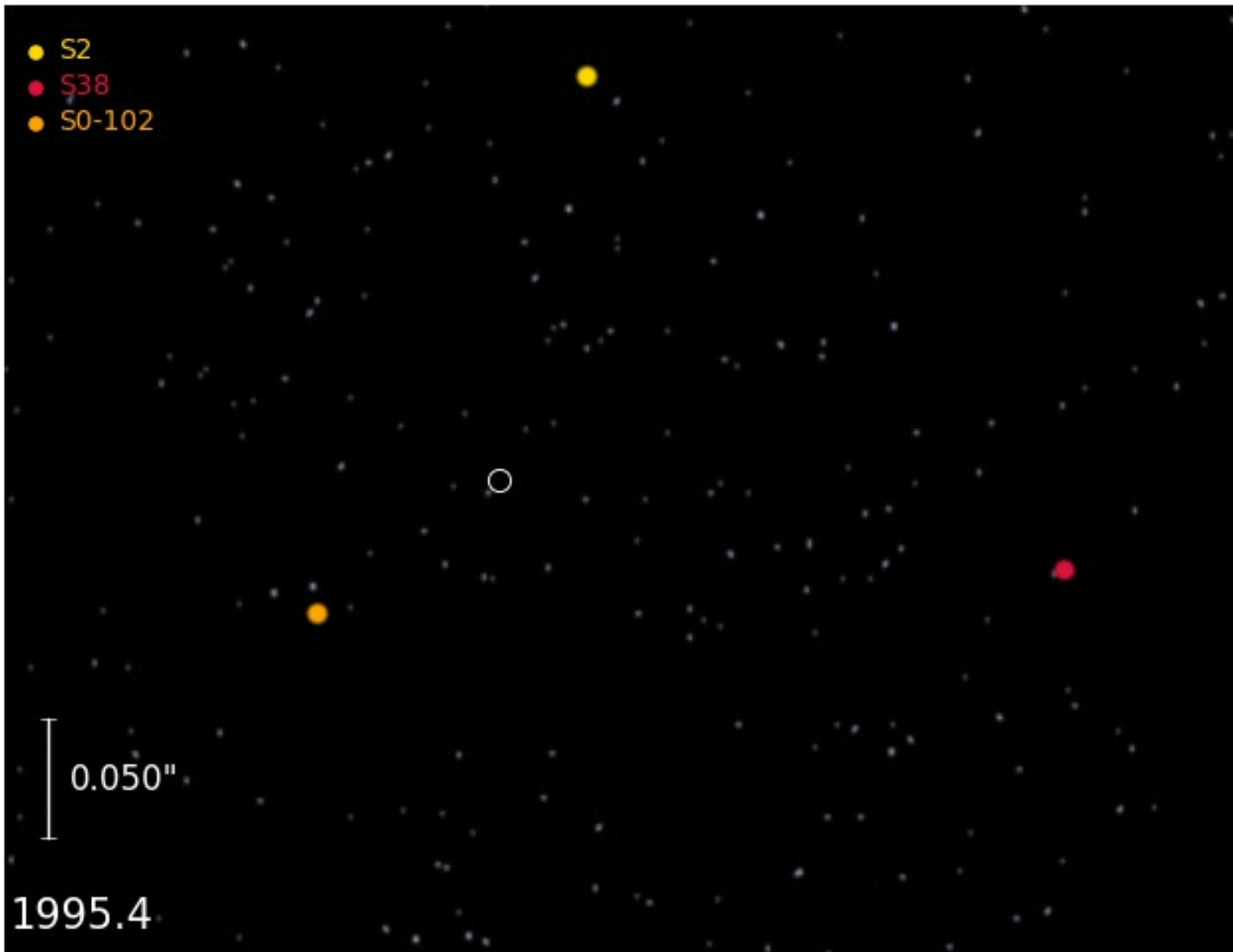


First relativistic analysis
using three stars orbiting SgrA*!

Relativistic distortion of orbits
is used to parameterize a
relativistic parameter
which becomes an observable

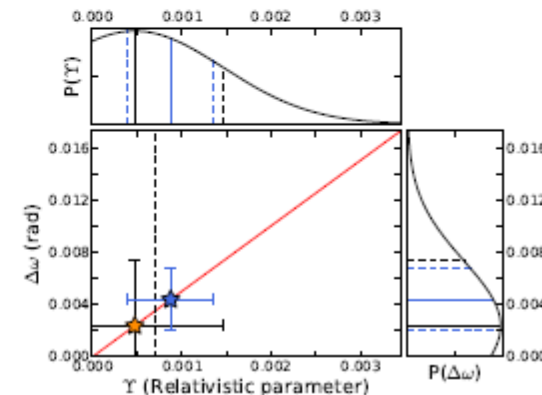
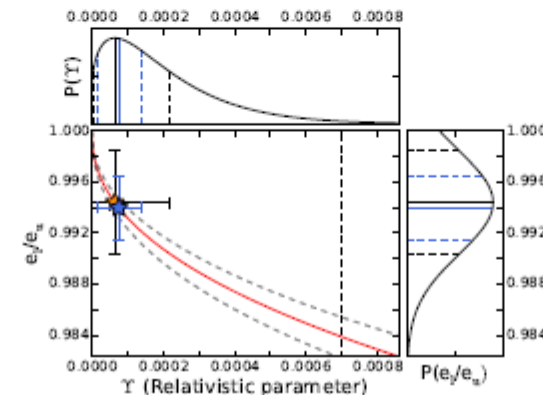
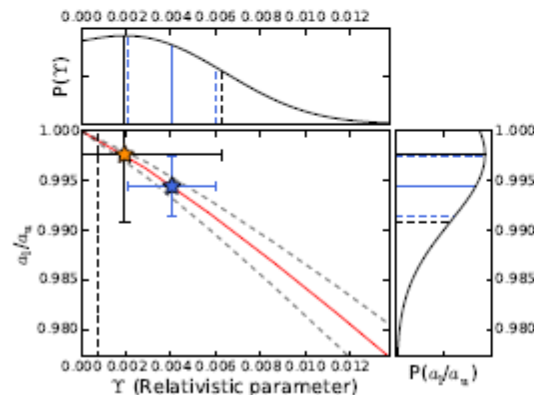
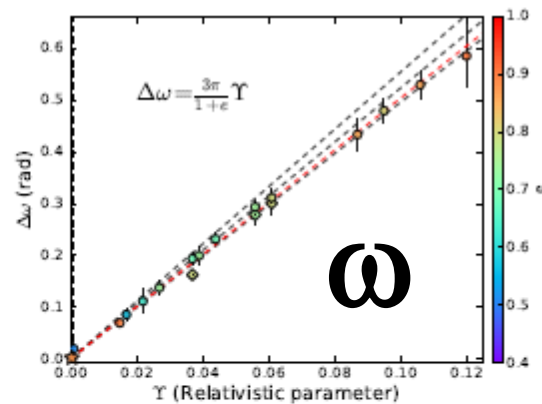
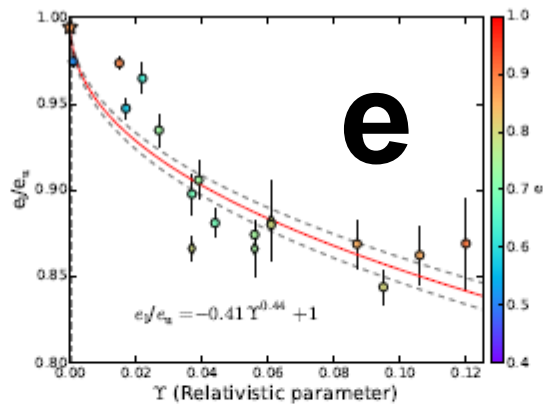
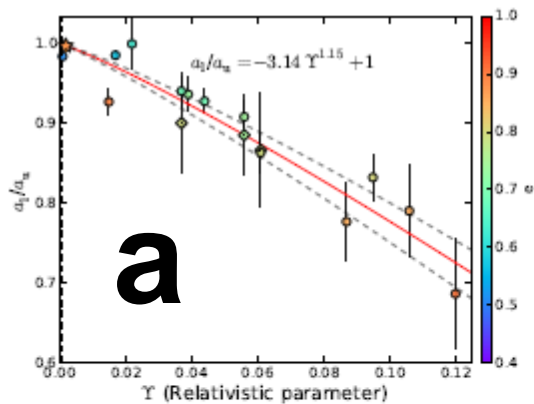


- S2
- S38
- S0-102



0.050"

1995.4



relativistic parameter defined as $\tilde{\Upsilon} \equiv r_s/r_p$

r_s Schwarzschild radius; r_p periaapse distance

expected value of $\tilde{\Upsilon} = 0.00065$

derived from M_{BH} and the orbit of S2

$$\tilde{\Upsilon} = 0.00088 \pm 0.00080$$

First time that the investigation of a resolved stellar orbit around an SMBH has been carried out in detail.

The result is consistent with the SMBH hypothesis.

For $\Delta\omega$ a 3-4 σ result

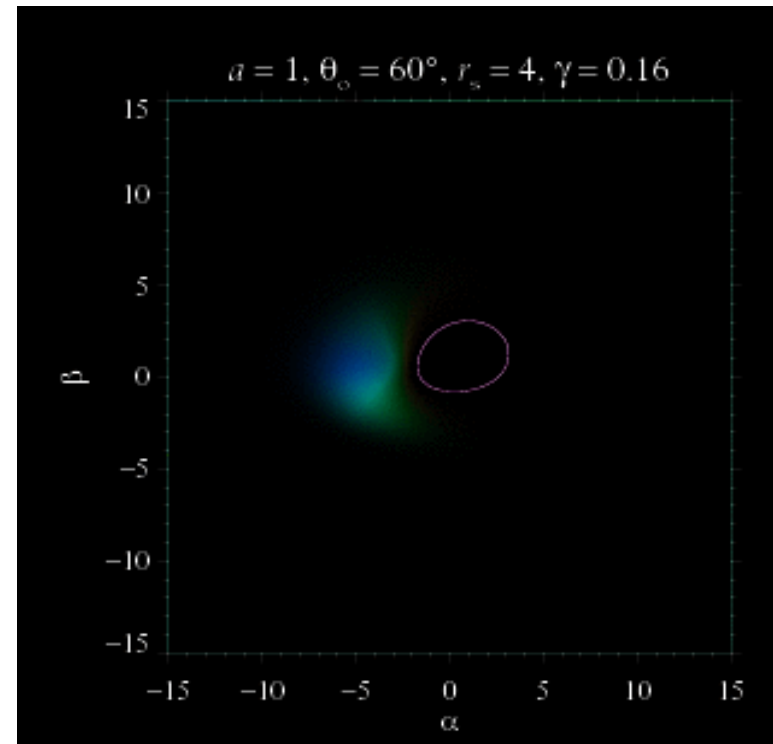
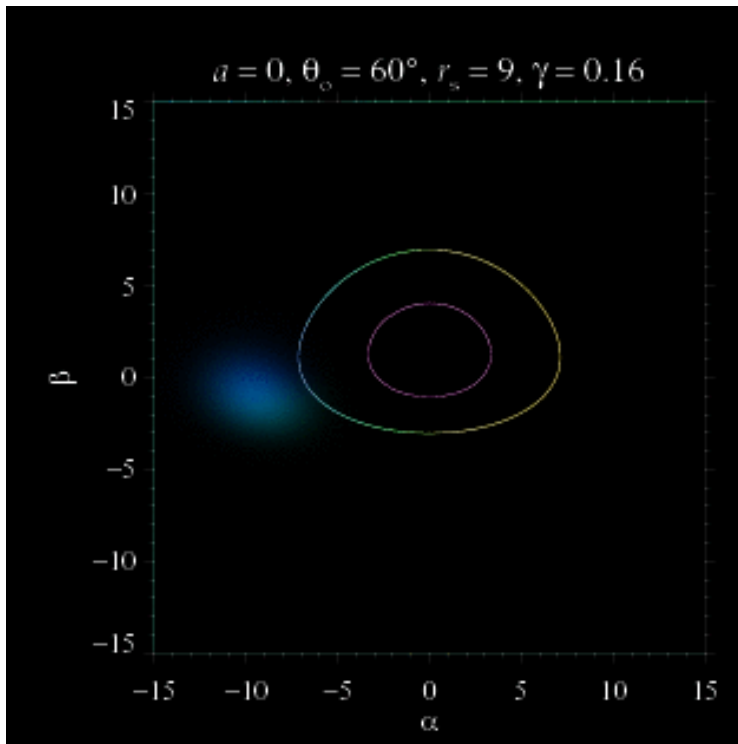
Example 2

Proving that we indeed probe a
relativistic regime:

Fitting flare profiles with blobs
moving close to the last stable orbit

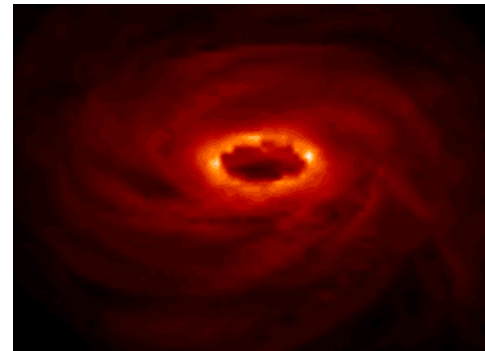
Karssen et al. 2017, MNRAS 472, 4422

Polarized Light from SgrA* in the Infrared



Dovciak, Karas & Yaqoob 2004, ApJS 153, 205
Dovciak et al. 2006

S. Karssen, M. Valencia-S., M. Bursa,
M. Dovciak, , V. Karas, A. Eckart



Analysis of 4 bright X-ray flares

observer



field
of
view

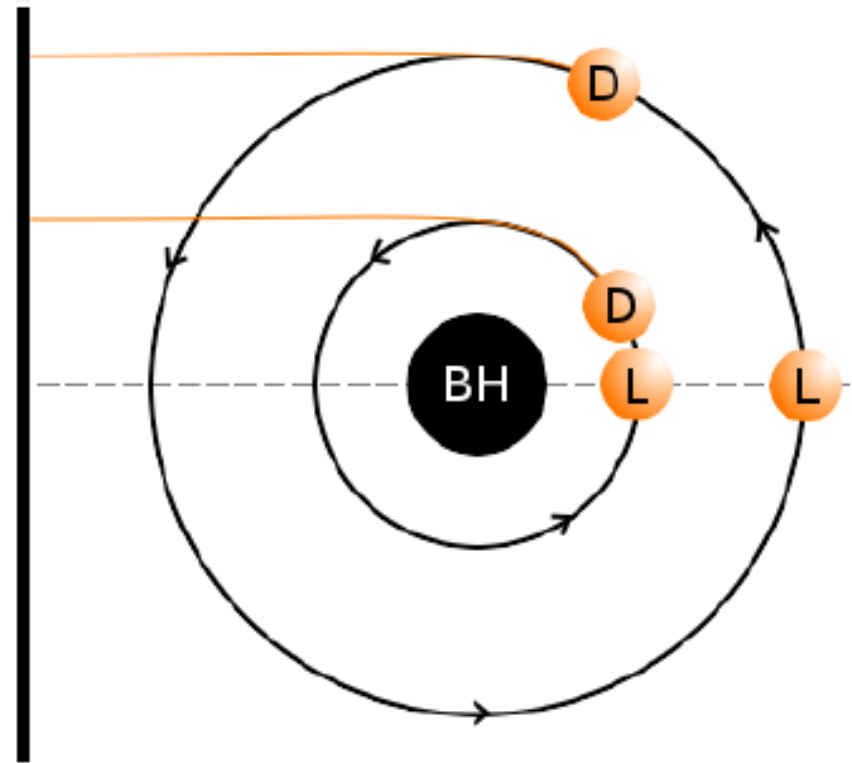
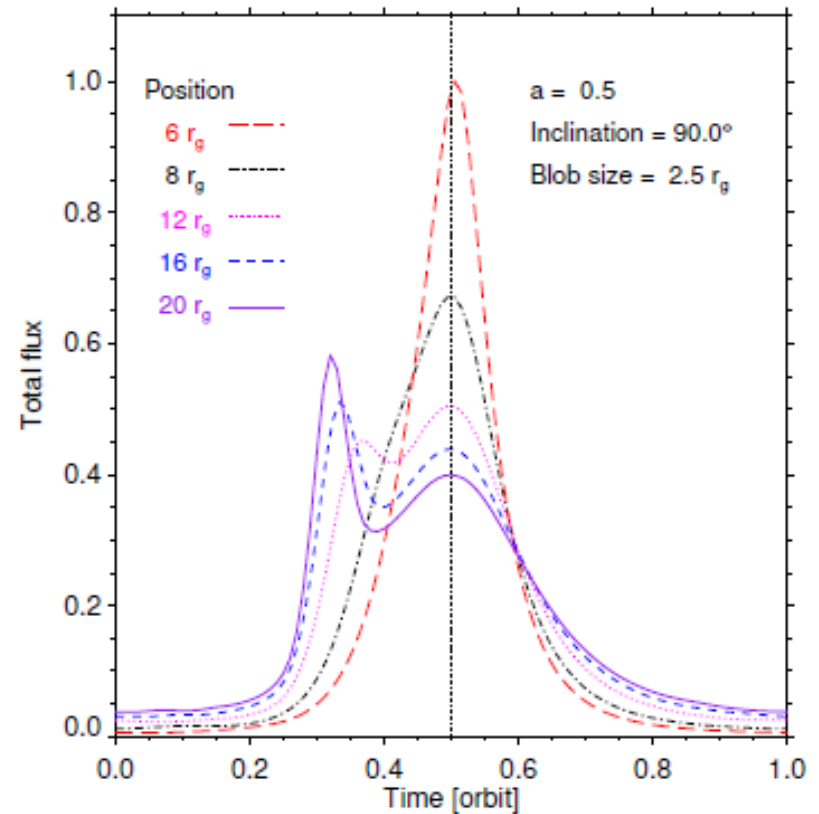
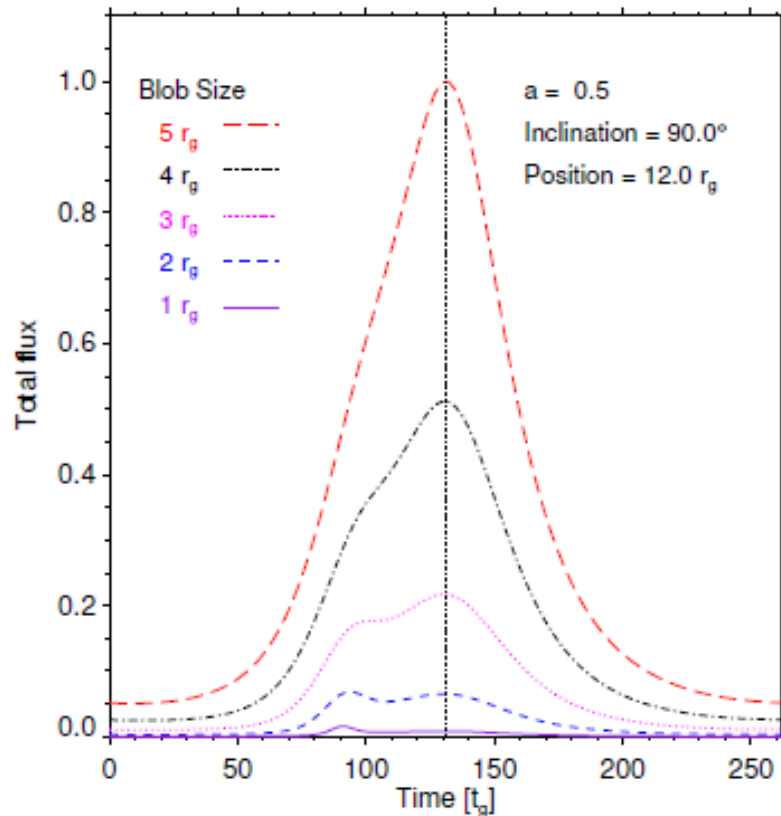


Fig. 1: Illustration of the origin of the double-peak structure in the total flux. The blobs marked with an 'L' are magnified by gravitational lensing, while they are behind the black hole from the observers point of view. That is, they are positioned on the focal line, as indicated by the dashed line. The blobs marked with a 'D' are Doppler-boosted, because they are moving 'directly towards' (in terms of geodesics) the observer, as indicated by the orange lines representing the geodesics from the source to the observer. The fraction of the orbit between these points varies with the radius of the orbit, owing to the stronger bending of the geodesics close to the black hole.

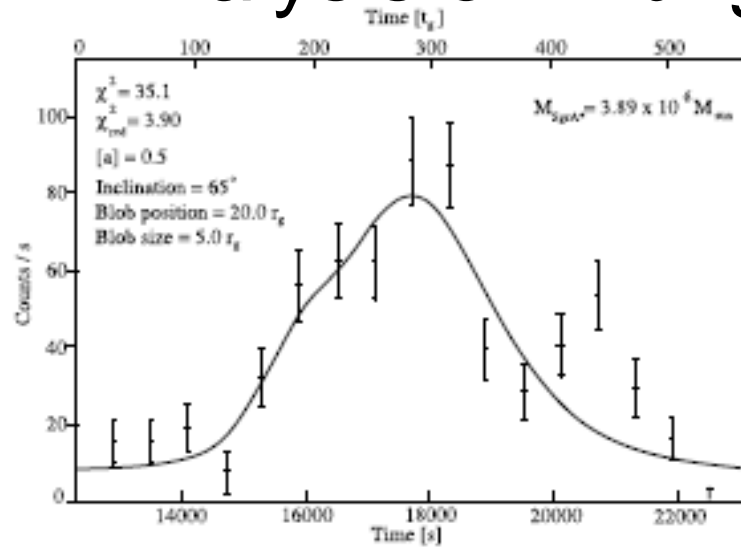
Analysis of 4 bright X-ray flares



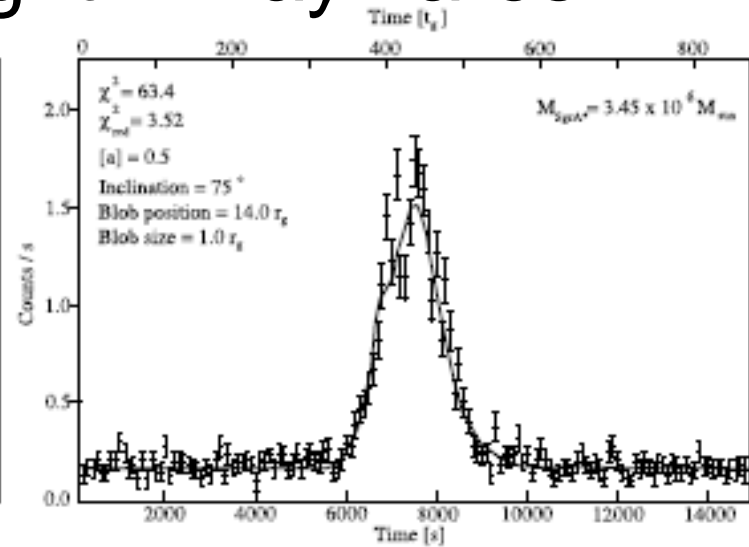
(a) Illustrates the influence of the blob's size on the shape of the light curve. The blobs of different sizes ($5 r_g$ red long dashed, $4 r_g$ black light curve, $3 r_g$ magenta dotted, $2 r_g$ blue short dashed and $1 r_g$ solid purple line) are orbiting at a radial position of $12 r_g$ around a black hole with spin 0.5, the viewing angle is 90° (edge on). The light curves are normalized to the maximum of the peak value of the light curve for the blob with the size $5 r_g$ and shifted such that the dopple peak is at the center.

(b) Illustrates the influence of the blob's position on the shape of the curve. The blobs are orbiting at different positions ($6 r_g$ red long dash-dotted, $8 r_g$ black dash-dotted, $12 r_g$ magenta dotted, $16 r_g$ blue short dashed and $20 r_g$ solid purple line) and have a size of $2.5 r_g$ around a black hole with spin 0.5, the viewing angle is 90° (edge on). The light curves are normalized to the maximum of the peak value of the light curve for the blob with the size $5 r_g$ and shifted such that the dopple peak is at the center.

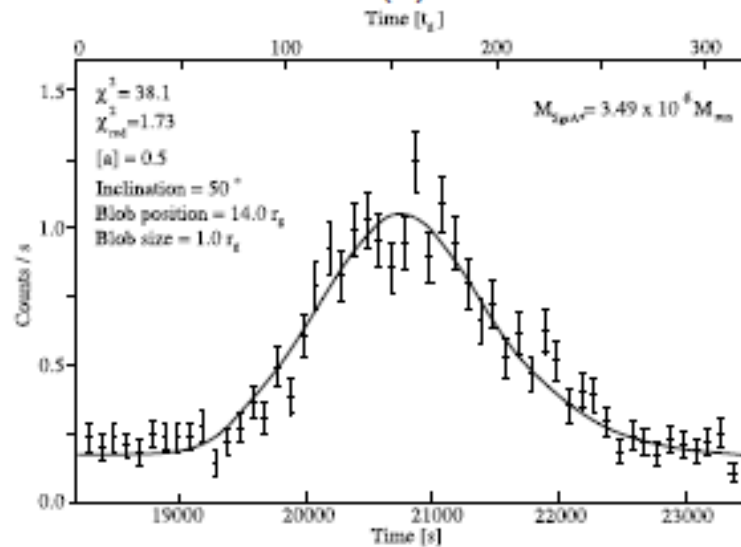
Analysis of 4 bright X-ray flares



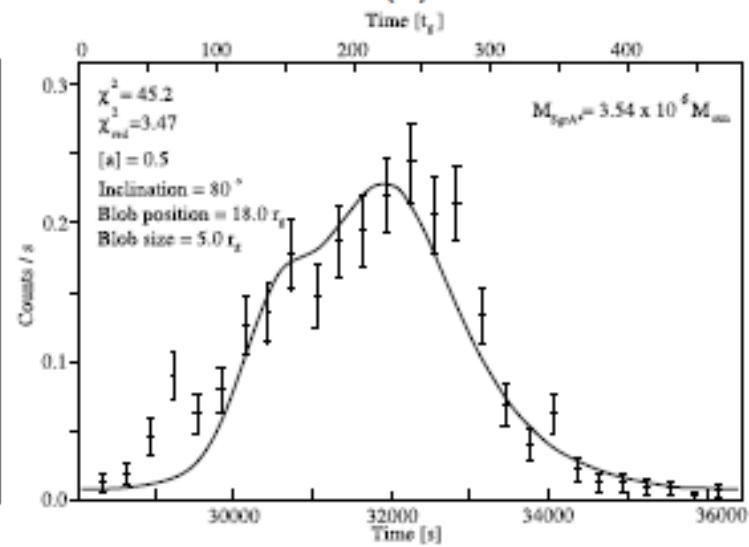
(a)



(b)

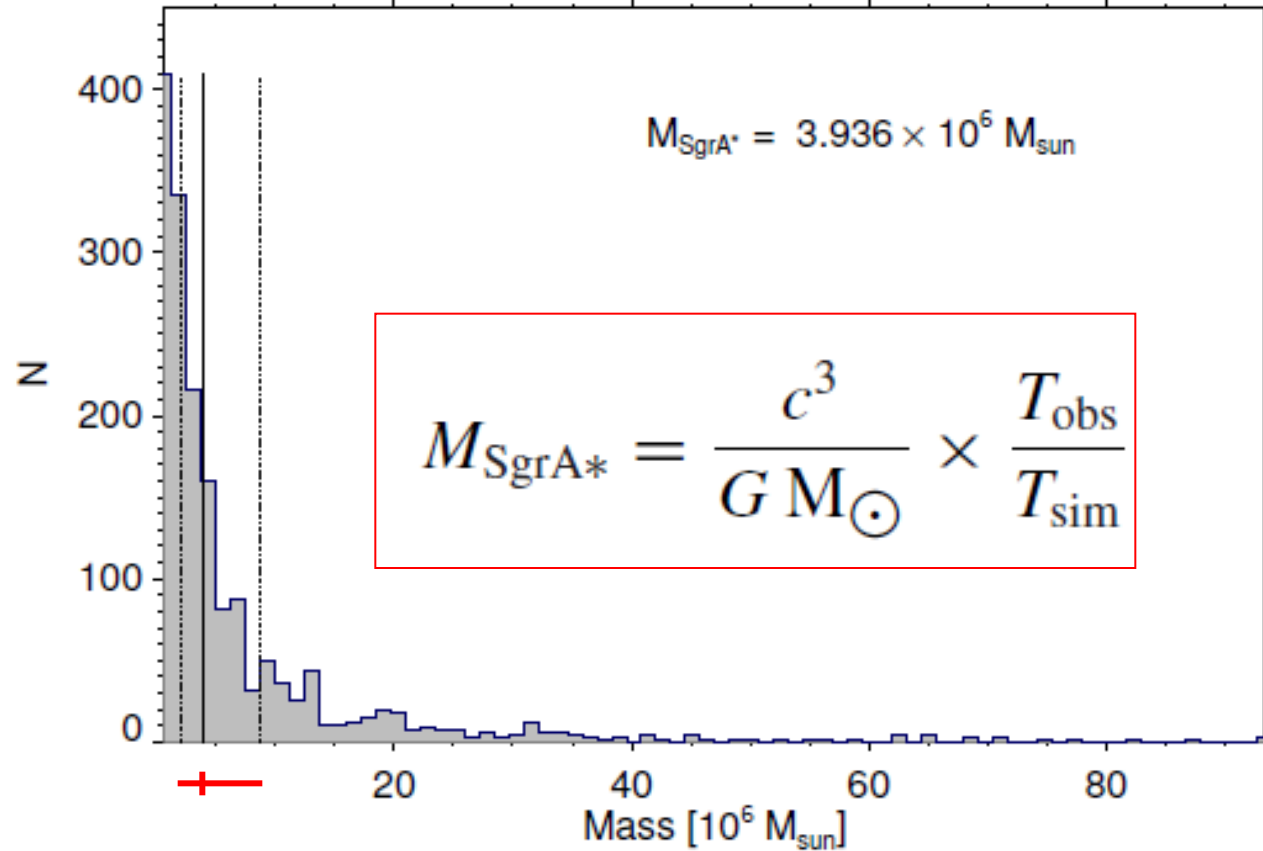


(c)



(d)

Analysis of 4 bright X-ray flares



(g) All flares together give a median mass of $3.936 \times 10^6 M_{\odot}$.

Fig. 2: Weighted histograms of the predicted masses for all the models for the four flares taken into account.

Application to a different extragalactic SMBH: J1034-396

Table 3. Mass estimates of the Seyfert I galaxy RE J1034+396 with different methods in a chronological order.

Publication	Mass	Method
Gierliński et al. (2008)	$6.3 \times 10^5 M_{\odot}$	H β
Gierliński et al. (2008)	$3.6 \times 10^7 M_{\odot}$	[O III]
Gierliński et al. (2008)	$(8 \times 10^6 - 9 \times 10^7) M_{\odot}$	ISCO
Bian & Huang (2010)	$(1-4) \times 10^6 M_{\odot}$	$M - \sigma_*$
Bian & Huang (2010)	$(1-4) \times 10^6 M_{\odot}$	H β
Jin et al. (2012)	$1.7 \times 10^6 M_{\odot}$	H β
This paper	$1.421 \times 10^6 M_{\odot}$	hotspot

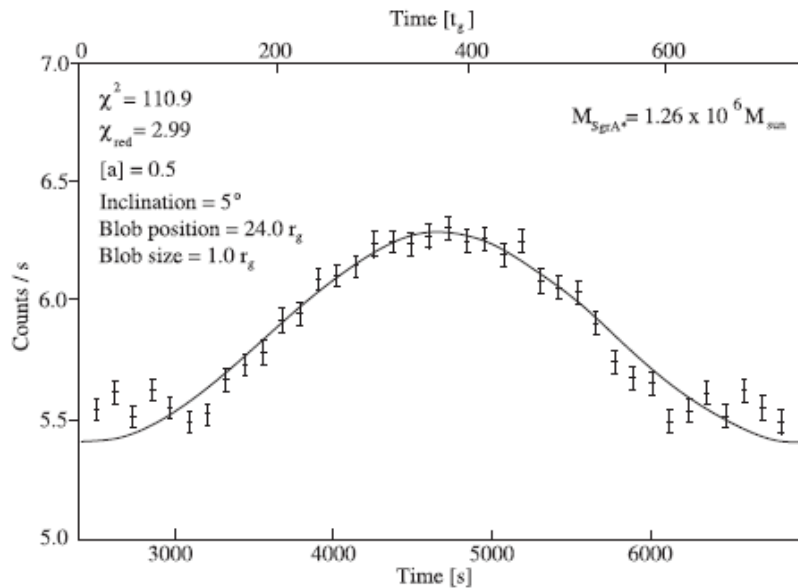


Figure 9. Best fit for the QPO of J1034-396 show for data of the folded light curve as published by Gierliński et al. (2008).

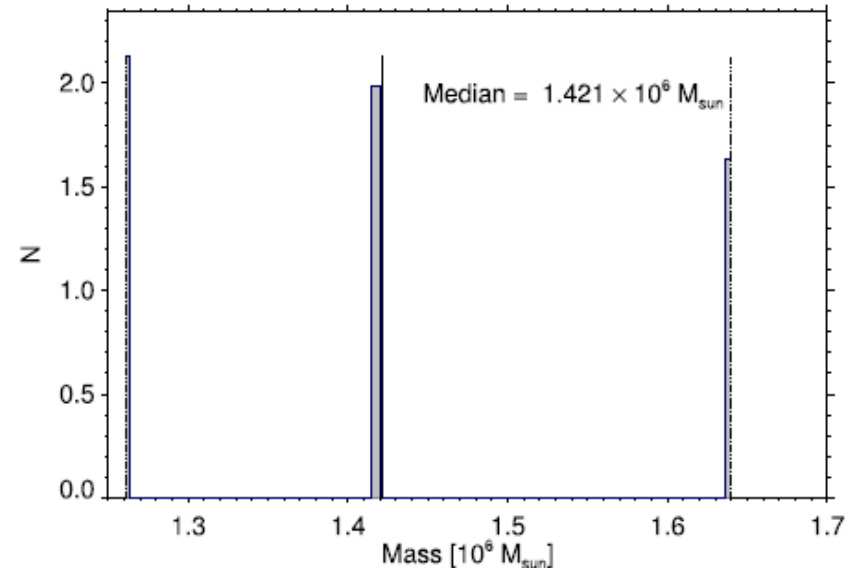


Figure 8. Weighted histogram for the QPO of J1034-396 as published by Gierliński et al. (2008).

Example 3

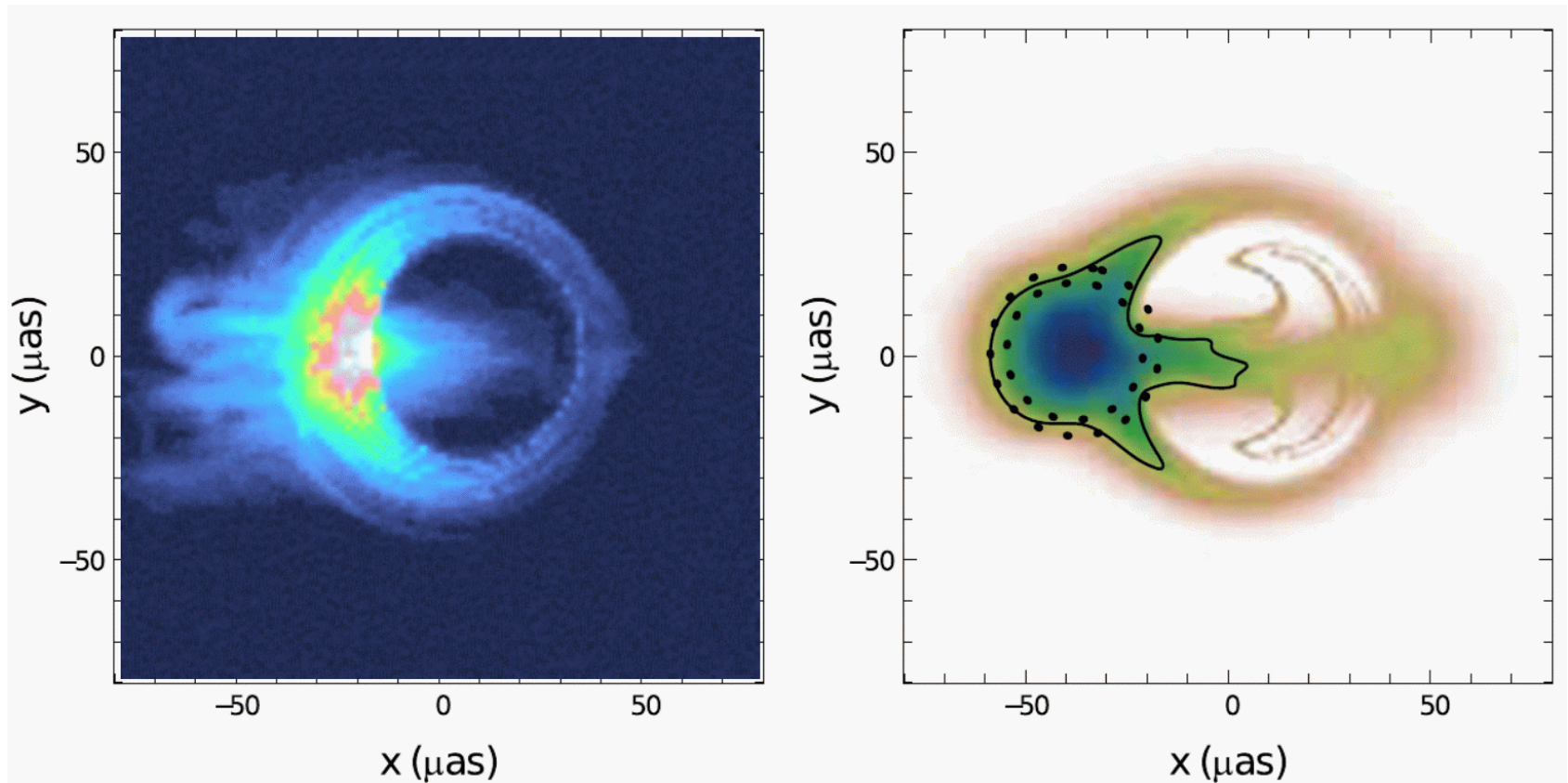
Toward the Event Horizon
Search for the Shadow of the Black Hole

VLBI (EHT) and VLTI (GRAVITY) interferometry

Rauch et al. 2016, *A&A* 587, 37

Eckart et al. *FoPh* 47, 553

The Shadow of the Black Hole



The shadow of the compact mass at the center of the Milky Way as expected for a **Black Hole (left)** and a **Boson star (right)** .

Goddi, C.; Falcke, H.; Kramer, M.; Rezzolla, L.; et al., 2017, IJMPD (International Journal of Modern Physics D), 2630001, BlackHoleCam: Fundamental physics of the galactic center

Vincent, F. H.; Meliani, Z.; et al., 2016, CQGra 33, 5015, Imaging a boson star at the Galactic center

Expected Photo-Center motion for SgrA*

Probably possible
with GRAVITY
at the VLTI

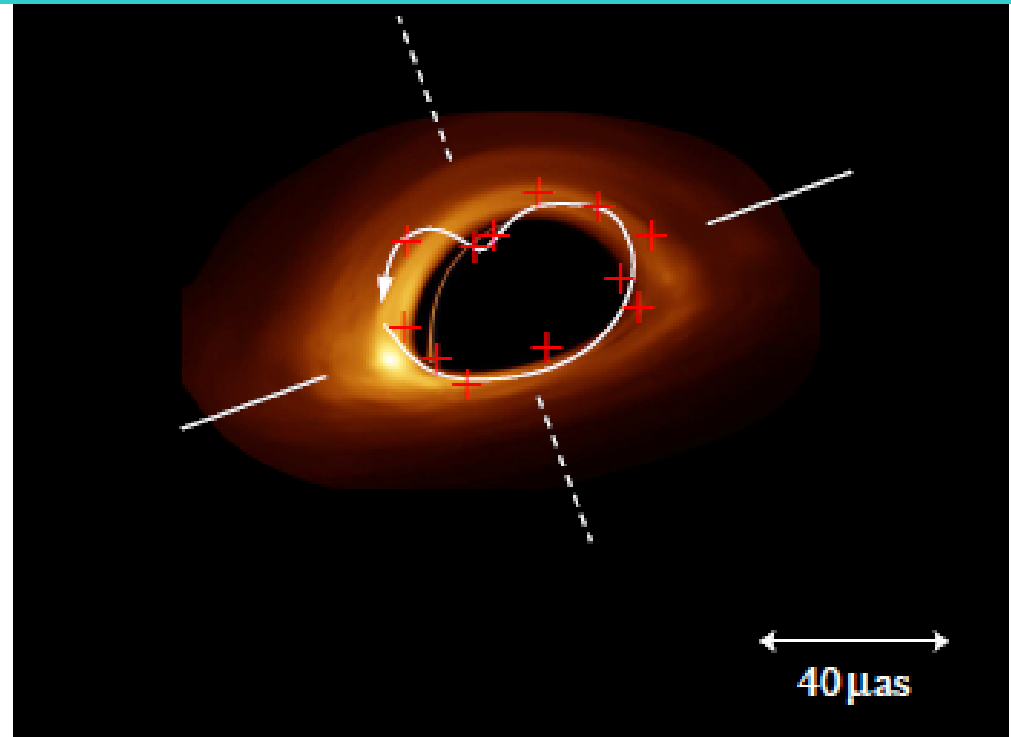
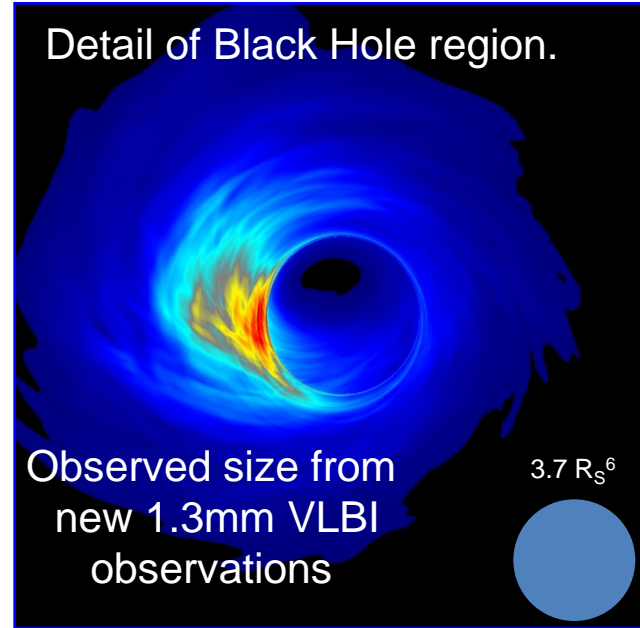
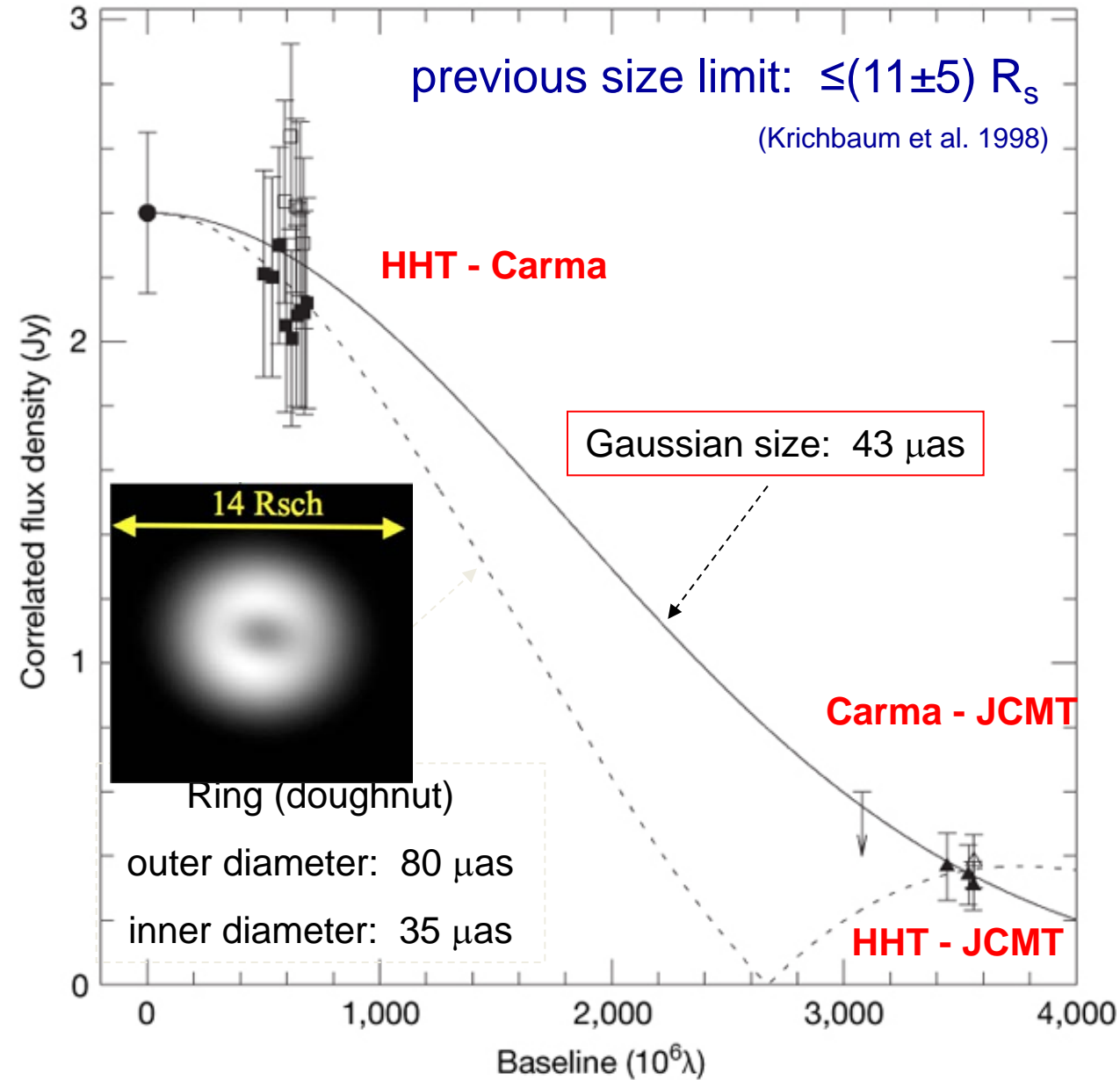


FIG. 10. Photocenter motion compared to a disk model. The example of a NIR photo-center motion as planned to be measured with the GRAVITY interferometer at the VLTI is taken from [256] and [257]. The simulation describes the apparent trajectory of flare events assuming material orbiting a non-rotation black hole at an inclination of 45° on the last stable orbit at a distance of $3 R_S$ from the center. Lensing (including multiple images), relativistic beaming and Doppler effect are included in the relative positioning of the resulting data points (red crosses) following the orbital track ([white line; further details in 256]). The image [162] is assumed to represent a mm-VLBI data disk model that shows luminous material for radii beyond the last stable orbit. The dashed and straight white arrows indicate the directions perpendicular and along the radio structure that we refer to in the text.

Eckart et al. FoPh 47, 553
and references there in

VLBI at 230 GHz (1.3 mm wavelength)



observed size:
 $43 (+14/-8) \mu\text{as}$
 deconvolved :
 $37 \mu\text{as} (3.7 R_s)$

Nature of some SgrA* radio flares

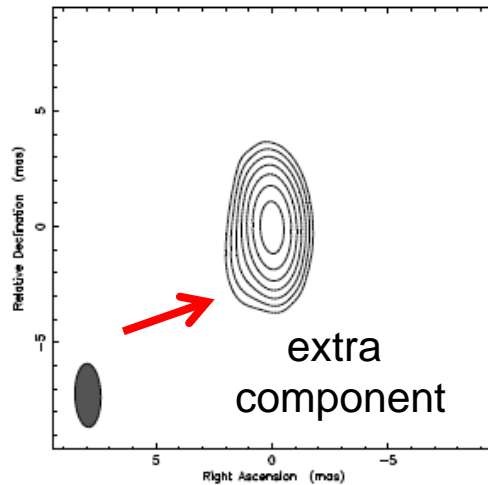


Fig. 7: RCP map of SgrA* on May 17 2012 (8-10h UT). The map was convolved with a beam of 2.74×1.12 at 1.76° . Contour levels are 1, 2, 4, 8, 16, 32 and 64% of the peak flux density of 1.5 Jy/beam.

Central component of 1.55 Jy
secondary component of 0.02 Jy
at 1.5 mas and 140 deg. E-N
with a 4 hour delay relative to the
NIR flare

Rauch et al. 2016, A&A 587, 37

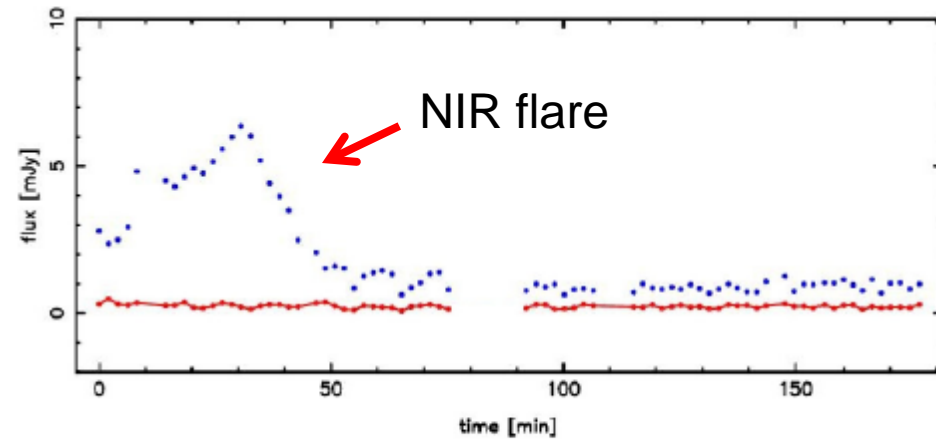


Fig. 3: NIR K_s-band ($2.2 \mu\text{m}$) light curve of Sgr A* observed in polarimetry mode on 17 May 2012. The light curve shown is produced by combining pairs of orthogonal polarization channels: 0° and 90° (taken from Shahzamanian et al. (2015)). Observations started at 4:55 AM UT.

Bower et al. (2014) report major axis sizes of SgrA* as an elliptical Gaussian of $35.4 \times 12.6 R_S$ at a position angle of 95° east of north. Which is much lower than the discussed source morphology due to a secondary component of 0.02 Jy at 1.8 ± 0.4 mas at 140° east of north.

See also 'Asymmetric structure in SgrA* ...'
Brinkerink et al. 2016, MNRAS 462, 1382
'speckle transfer function'

Example 4

Towards the Event Horizon
using stars and pulsars

Psaltis D., Wex N., Kramer M., 2016, A Quantitative Test of the No-hair Theorem with Sgr A*; Using Stars, Pulsars, and the Event Horizon Telescope. ApJ 818, 121

Eckart et al. FoPh 47, 553

Number of Stars within the Central 1000 AU of SgrA*

TABLE I. Number of stellar objects within 1000 AU of SgrA*

γ	N_{stars}	N_{msP}	N_{nP}
2.0	5000	5	0.5
1.2	67	0.67	0.067
1.0	24	0.24	0.024
2.0	6	-	-
1.2	0.08	-	-
1.0	0.03	-	-

N → a few to 0

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Approximate number of stars, millisecond pulsars msP , and normal pulsars nP with distances to SgrA* of less than 1000 AU. This corresponds to a radius of 0.125" or 4.7 mpc.

Using a value of $M_* = 10^6 M_\odot$ for the central parsec we derive for different values of γ the number of solar mass stars (second column in the top three rows) and stars with a $2\mu\text{m}$ wavelength brightness in the magnitude interval K=18-19 (second column in the bottom three rows). Using the estimate of 100 normal and 1000 millisecond pulsars within the central parsec [108, 116] we derived the corresponding values for the central 1000 AU in columns 3 and 4.

Synthesis:

Combining the
Necessary Conditions
to Sufficient Conditions

Necessary Conditions for the presence of a Black Hole

label	necessary condition
N_1	Is object at nominal position of SgrA*?
N_2	Is size of emitting region in SgrA* sufficiently small?
N_3	Is mass of SgrA* in agreement with SMBH masses?
N_4	Does the distance to SgrA* place it at the center of the Milky Way?
N_5	Is the manipulative success for SgrA* similar to other SMBH candidates?
N_6	Is a bright fast jet originating from SgrA*?
N_7	Do we detect a merger ringing signal in gravitational waves from SgrA*?
N_8	Do we detect an exceptionally bright flare from SgrA*?
N_9	Do stars and pulsars close to SgrA* give indications for a SMBH?
N_{10}	Is the spectrum of the surroundings of SgrA* what es expect from a SMBH?
N_{11}	Do we detect a photon ring in SgrA* in addition to orbiting matter?
N_{12}	Do VLBI images of SgrA* show a shadow as expected for a SMBH?
N_{13}	Do we detect photo-center motion of SgrA* with NIR- and/or mm-radio-interferometry?
N_{14}	Can we differentiate fo SgrA* between jet components and hot-spot?

Table 3 Table of possible necessary conditions that can be combined to result in a sufficient condition required to call SgrA* a SMBH. The necessary conditions have been formulated as logical entities for which we can attribute the locigal values “true” or “false” within the theoretical predictions for supermassive black holes in section 2.

Philosophical Concepts layed out for the GC

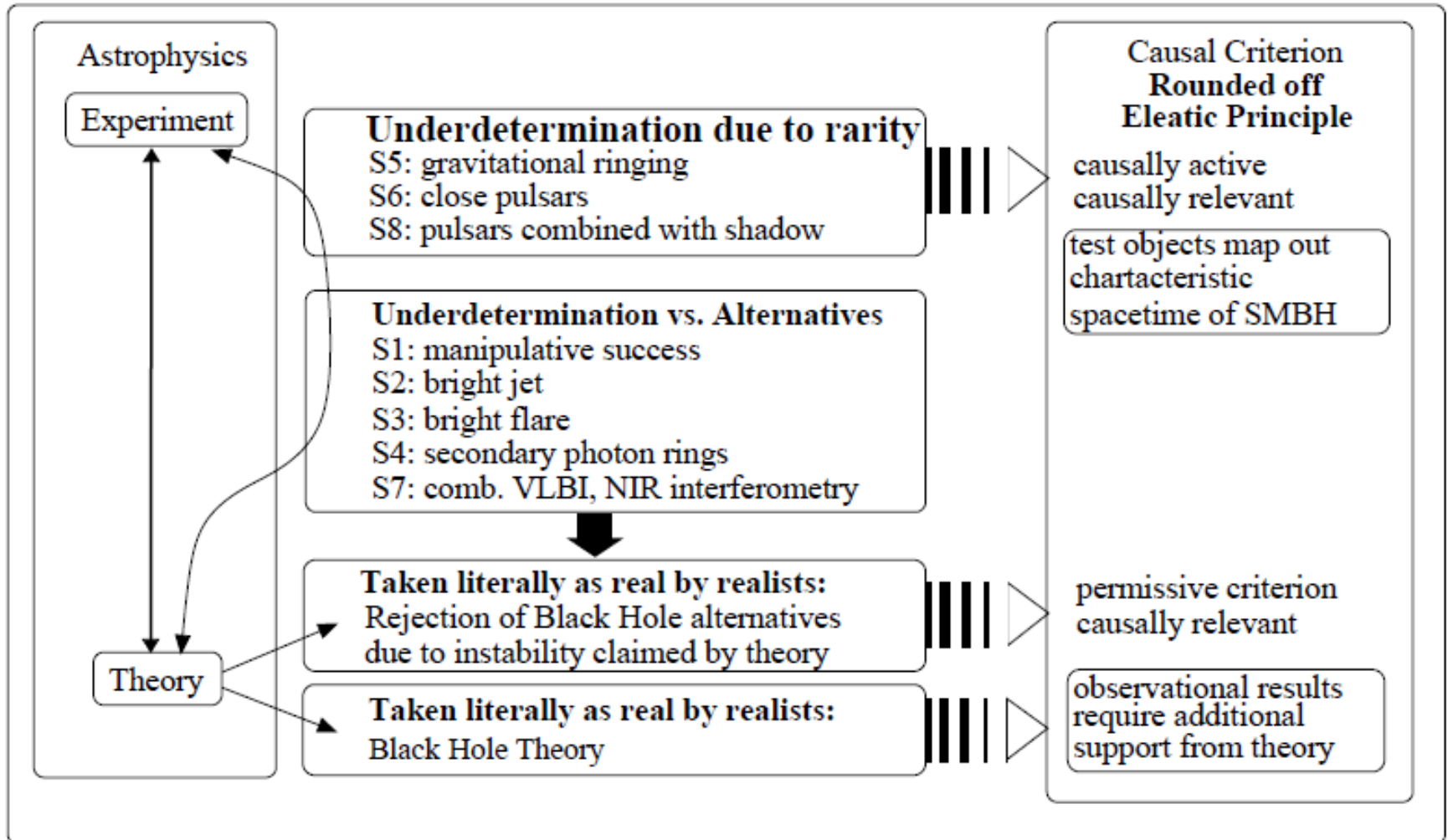


Fig. 13 Linkage between experiment and theory interpreted via the concept of realism, underdetermination and a “rounded out” version of the Eleatic Principle, here shown with respect to the results of our investigation. For comparison see also Fig.1 which we adopted here for the case of the Galactic Center SMBH.

Philosophical Concepts layed out for the GC

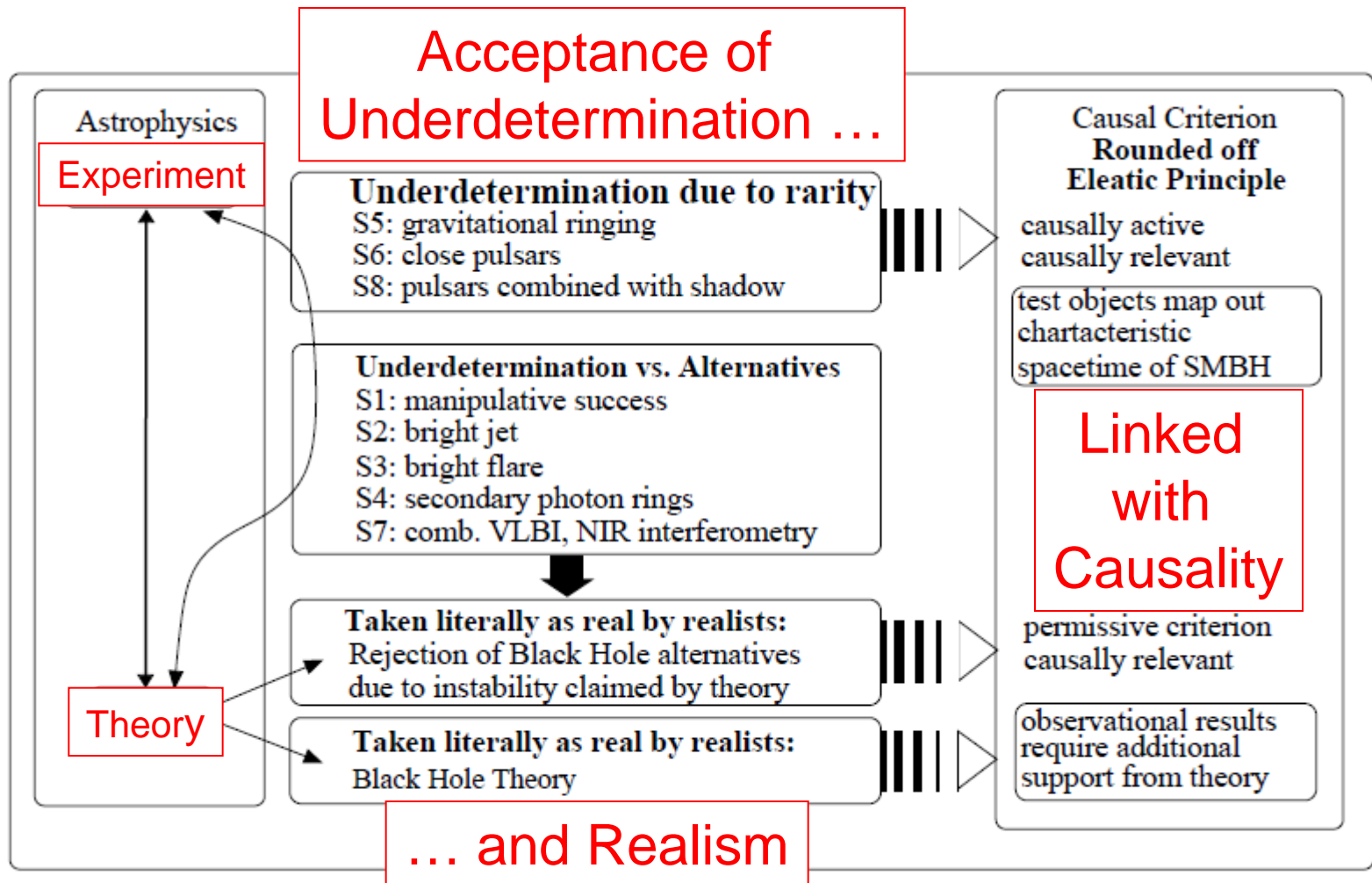
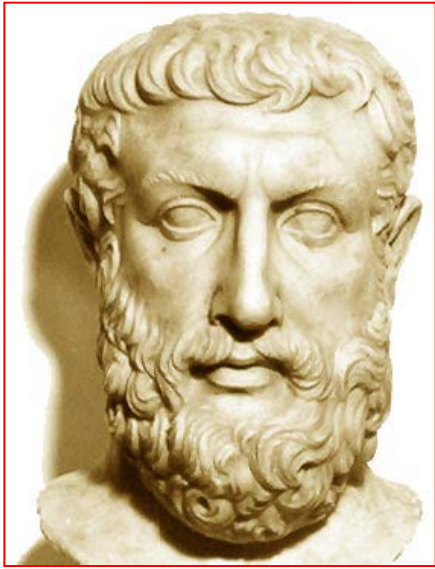


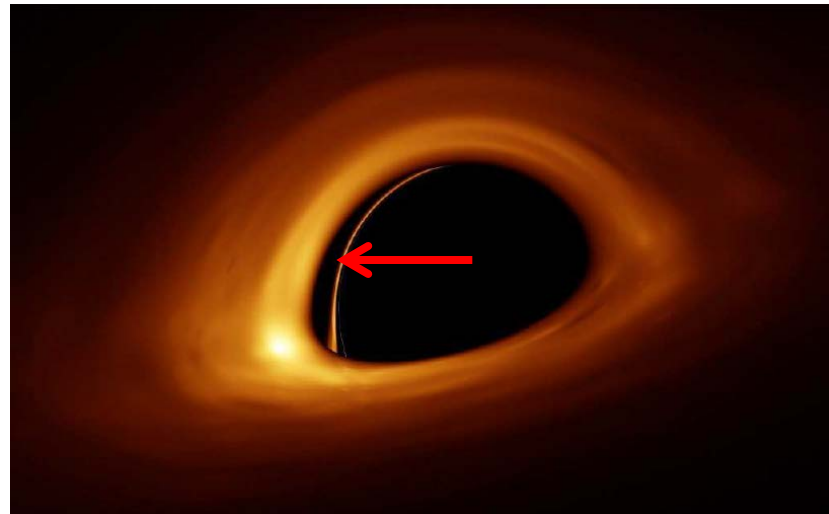
Fig. 13 Linkage between experiment and theory interpreted via the concept of realism, underdetermination and a “rounded out” version of the Eleatic Principle, here shown with respect to the results of our investigation. For comparison see also Fig.1 which we adopted here for the case of the Galactic Center SMBH.

Combining all results



Parmenides

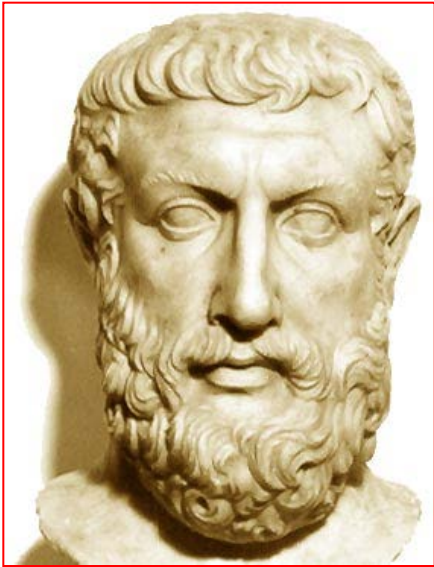
Combining the observational facts using a causal criterion test **may indeed lead to well supported confirmation** that SgrA* at the center of the Milky Way can be identified with a super massive black hole.



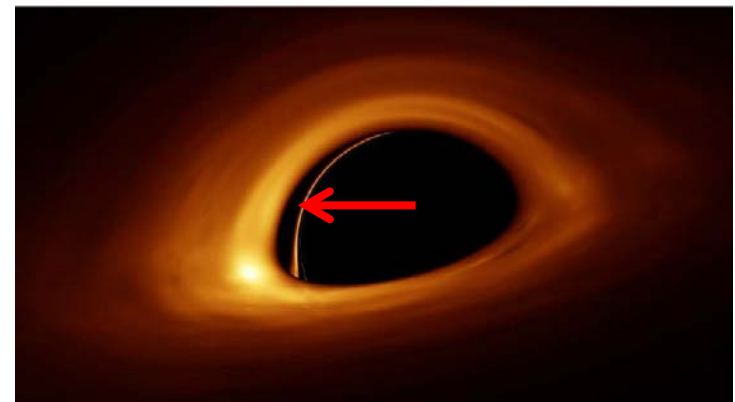
Combining all results

Challenge for Astrophysics: How clean are the observational cases that may serve as logical entities for the causal criterion test.

Challenge for Philosophy: Are all necessary conditions for the proof of existence known and fulfilled? Is the result a sufficient condition for the existence? Are there individual sufficient conditions that can proof the existance and are they risky enough?



Parmenides



Philosophical Concepts layed out for the GC

Terrible reality:

The 'easier' a key observation can be made
the less meaningful and stringent it is.
(Radio and infrared interferometry; shadow of the BH)

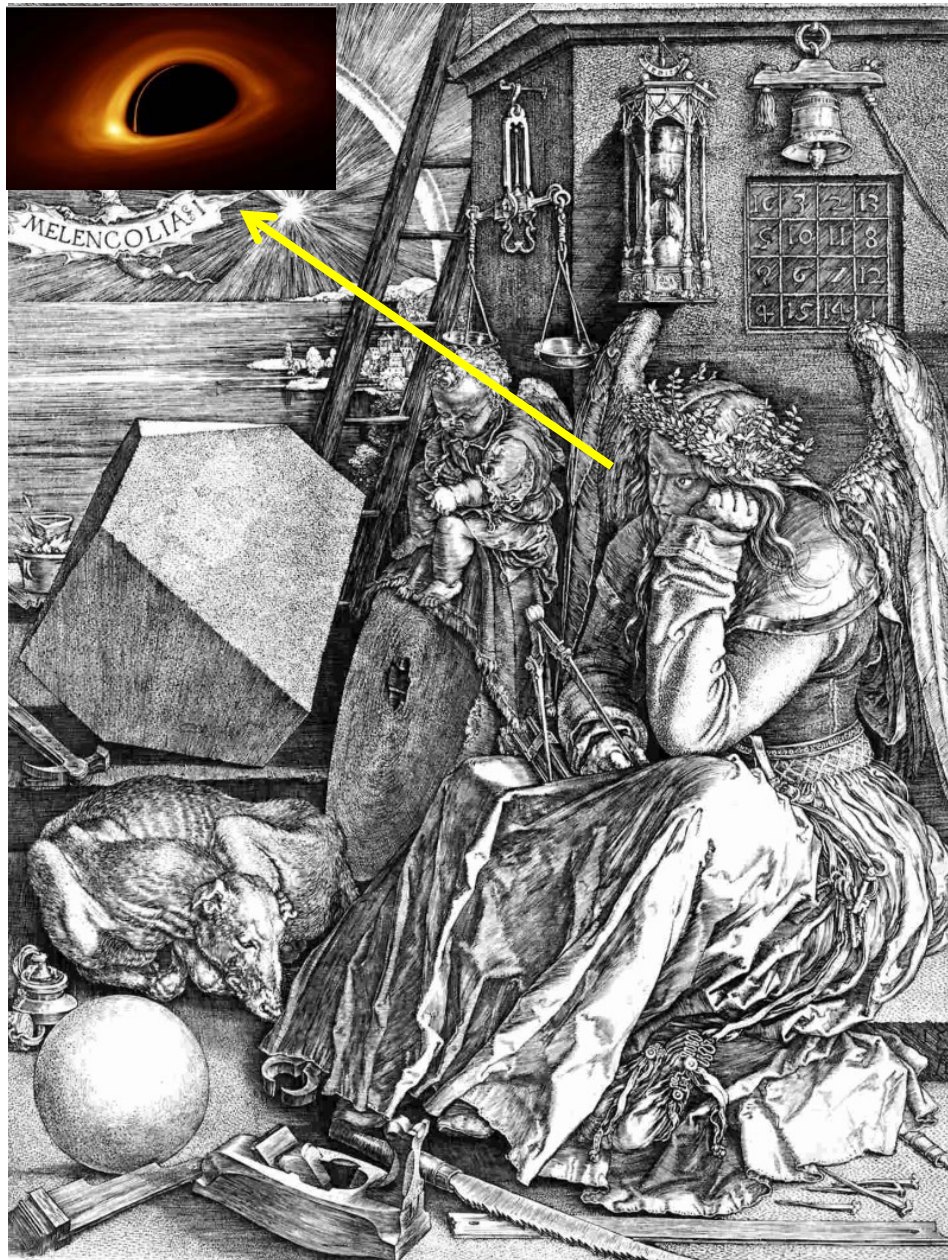
The more meaningful and stringent key observations
are the more difficult and rare they are.
(Pulsar and stellar measurements; gravitational waves)

**A real prove dosen't seem to be possible, however,
the acceptance of the idea can be maximized.**



'Melancholia' by
Albrecht Dürer

END



'Melancholia' by Albrecht Dürer

Are all necessary conditions for
the proof of existence known and fulfilled?
Is the result a sufficient condition for
the existence?

Is there at least one or are there several
sufficient conditions for the existence?

END

